For the following questions, specify which examination they should be in (Exam 1 – Tech free or Exam 2 – Tech Active) then allocate the number of marks and show the marking scheme.

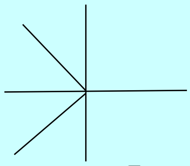
Question 1 Exam 1

Solve $2\cos(2x) = -\sqrt{3}$ for x, where $0 \leq x \leq \pi$.

$$\cos(2x) = -\frac{\sqrt{3}}{2}$$

$$2x = \frac{5\pi}{6}, \frac{7\pi}{6}$$

$$\therefore x = \frac{5\pi}{12}, \frac{7\pi}{12}$$



B.A. = $\frac{\pi}{6}$

1M – Recognise the basic angle is $\frac{\pi}{6}$

or any correct relative angle to $\frac{\pi}{6}$

1A – both correct answers.

Max 1 mark for correct answers with any incorrect notation/working in the solutions such as $\cos(2x) = \frac{5\pi}{6}, \frac{7\pi}{6}$

Question 2 Exam 1

a. Given that $(2x + 1)$ is a factor of $P(x) = 2x^3 - 9x^2 + kx + 6$, show that $k = 7$.

If $(2x + 1)$ is a factor of $P(x)$ then $P\left(-\frac{1}{2}\right) = 0$

$$2\left(-\frac{1}{2}\right)^3 - 9\left(-\frac{1}{2}\right)^2 + k\left(-\frac{1}{2}\right) + 6 = 0$$

$$-\frac{1}{4} - \frac{9}{4} - \frac{k}{2} + 6 = 0$$

$$-\frac{5}{2} + \frac{12}{2} = \frac{k}{2}$$

$$\therefore k = 7$$

1A – Evidence of using factor theorem to solve for k.

Accept subs $k = 7$ and show that $P\left(-\frac{1}{2}\right) = 0$.

Provide an advice if students use this subs method.

b. Hence, fully factorise $2x^3 - 9x^2 + 7x + 6$.

$(2x + 1)(x^2 - 5x + 6)$

$\therefore (2x + 1)(x - 2)(x - 3)$

1M – A valid approach in finding a quadratic factor. Eg: using a synthetic division, long division (not on course but acceptable), need to see $(x^2 - ax + 6)$

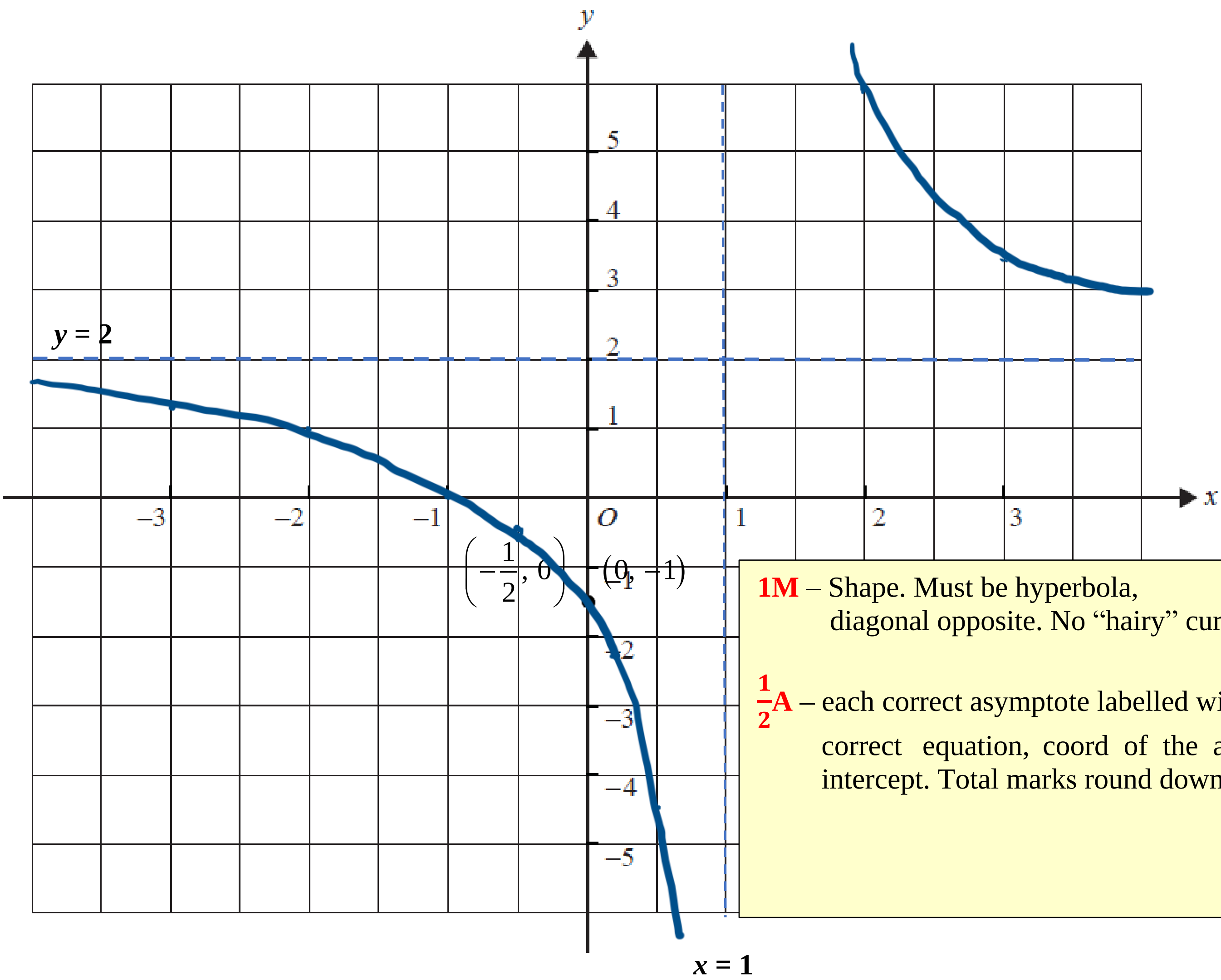
1A – All 3 correct linear factors.

0A – $x = -\frac{1}{2}$, 2, 3

Question 3 **Exam 1 can also be in Exam 2**

Let $f: R\backslash\{1\} \to R$, where $f(x) = 2 + \dfrac{3}{x-1}$.

a. Sketch the graph of f. Label any axis intercepts with their coordinates and label any asymptotes with the appropriate equation.



1M – Shape. Must be hyperbola, diagonal opposite. No "hairy" curve.

$\frac{1}{2}$**A** – each correct asymptote labelled with correct equation, coord of the axis intercept. Total marks round down.

b. Find the area enclosed by the graph of f, the lines $x = 2$ and , and $x = 4$ the –axis.

$$\text{Area } = \int_2^4 2 + \frac{3}{x-1}\,dx$$

$$= \left[2x + 3\log_e(x-1)\right]_2^4$$

$$= 2(4) + 3\log_e(4-1) - (2(2) + 3\log_e 1)$$

$$\therefore \text{Area } = 4 + 3\log_e(3)$$

This is a Y12 Exam question

1M – A definite integral, must have log and terminals of 2 and 4 in any order.

1A – Correct answer, accept $4 + \log_e 27$ or $4 + \log_e 3^3$

Question 4 **Exam 2**

The temperature, T °C, in an office is controlled. For a particular weekday, the temperature at time t, where t is the number of hours after midnight, is given by the function

$$T(t) = 19 + 6\sin\left(\frac{\pi}{12}(t-8)\right),\ 0 \le t \le 24.$$

a. For how many hours of the day is the temperature greater than or equal to 19 °C?

$T(t) = 19$ at $t = 8, 20$

$\therefore T(t) \ge 19$ for 12 hours

1M – $T(t) = 19$ or $T(t) \ge 19$ or $t = 8, 20$ is seen
1A – Correct answer. Unit is not required.

$f(t):=19+6\cdot\sin\left(\frac{\pi}{12}\cdot(t-8)\right)$ Done

$\text{solve}(f(t)=19,t)|0\le t\le 24$ $t=8$ or $t=20$

b. What is the average rate of change of the temperature in the office between 8.00 am and noon?

$\dfrac{T(12)-T(8)}{12-8}$ or $\dfrac{3\sqrt{3}+19-19}{12-8}$

$=\dfrac{3\sqrt{3}}{4}$

1M – apply the average rate of change formula correctly.

1A – Correct answer, unit is not required.

$\dfrac{f(12)-f(8)}{12-8}$ $\dfrac{3\cdot\sqrt{3}}{4}$

c. **i.** Find $T'(t)$.

$\dfrac{\pi}{2}\cos\left(\dfrac{\pi}{12}(t-8)\right)$

or

$-\dfrac{\pi}{2}\cos\left(\dfrac{\pi}{12}t+\dfrac{\pi}{3}\right)$

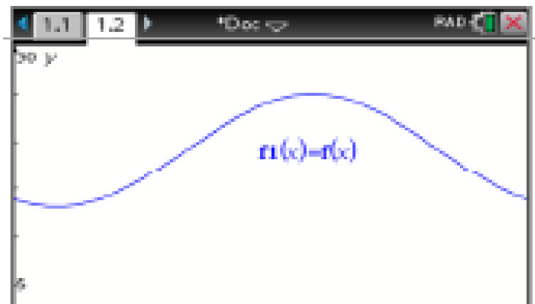
1A – Correct answer
Accept t or x

ii. At what time of the day is the temperature in the office decreasing most rapidly?

$T'(t) = -\dfrac{\pi}{2}$

$\therefore t = 20$

$\therefore$ 8 pm or 20:00



1M – A valid method that results in the greatest rate of decrease, as shown in the graph. Accept graph.

1A – Correct answer

$\text{solve}\left(\frac{d}{dt}(f(t))=\frac{-\pi}{2},t\right)|0\le t\le 24$ $t=20$

Question 5 **Exam 2**

Let $g: R \to R,\ g(x) = x^4 - 8x.$

a. Find the equation of the tangent to the graph of $y = g(x)$ at the point $(p, g(p))$.

$y = 4(p^3 - 2)x - 3p^4$

1A – Correct answer, must be an equation.

$y = \text{tangentLine}(x^4 - 8\cdot x, x, p)$

$y = 4\cdot(p^3 - 2)\cdot x - 3\cdot p^4$

b. Find the equations of the tangents to the graph of $y = g(x)$ that pass through the point with coordinates $\left(\frac{3}{2}, -12\right)$.

$y = 4(p^3 - 2)x - 3p^4$ passes through $\left(\frac{3}{2}, -12\right)$

$-12 = 4(p^3 - 2)\left(\frac{3}{2}\right) - 3p^4$

$\therefore\ p = 0,\ 2$

Equations of the tangents are:
$y = -8x$ and $y = 24x - 48$

1M – A valid approach to find the values of p
or
$p = 0,\ 2$ is seen.

1A– Each correct equation.

$\text{solve}(-12 = 4\cdot(p^3 - 2)\cdot x - 3\cdot p^4, p) | x = \frac{3}{2}$

$p = 0 \text{ or } p = 2$

$y = \text{tangentLine}(x^4 - 8\cdot x, x, p) | p = \{0, 2\}$

$y = \{-8\cdot x, 24\cdot x - 48\}$

Question 6 **Exam 1**

Let $f: R \setminus \{-2\} \to R,\ f(x) = \dfrac{2x+1}{x+2}$

Find the rule and domain of f^{-1}, the inverse function of f .

For inverse function, swap x and y:

$x = \dfrac{2y+1}{y+2}$

$x(y+2) = 2y+1$

$xy - 2y = 1 - 2x$

$y(x-2) = 1 - 2x$

$y = \dfrac{1-2x}{x-2}$

$\therefore\ f^{-1}(x) = \dfrac{1-2x}{x-2}$ or $f^{-1}(x) = \dfrac{2x-1}{2-x}$ or $f^{-1}(x) = \dfrac{-3}{x-2} - 2$

domain = $R \setminus \{2\}$

1M – swap x and y and a good attempt of solving for y.

1A – Correct inverse rule.
Do not accept y = or $f^{-1} = \ldots$

1A – Correct domain

Question 7

Solve $\log_3(x)-\log_3\left(2\left(x^2-9\right)\right)=-2$ for x.

$$\log_3\left(\frac{x}{2\left(x^2-9\right)}\right)=-2$$

$$\frac{x}{2\left(x^2-9\right)}=3^{-2}$$

$$9x=2\left(x^2-9\right)$$

$$2x^2-9x-18=0$$

$$(2x+3)(x-6)=0$$

$$\therefore x=-\frac{3}{2},\ 6$$

$$\therefore x=6$$

1M – A correct use of log law

1H – Form a quadratic equation and solve their equation correctly.

1A – Correct answer